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DYNAMIC MEAN–VARIANCE OPTIMIZATION OF PORTFOLIOS USING QUADRATIC PROGRAMMING TECHNIQUE.

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ABSTRACT

In this study, the quadratic programming (QP) technique is applied in portfolio optimization using the Markowitz mean-variance model. The portfolio selection problem considered equity market indices for the developed exchanges in the world – New York stock Exchange (NYSE), National Association of Securities Dealers Automated Quotations stock exchange (NASDAQ), Shanghai stock exchange (SSE) and Japanese stock exchange (JPX)). Using daily log returns data of each of these stock exchanges from January 2025 to March 2026, optimal portfolio selection for cases of 2-asset, 3-asset and 4-assets portfolio investments under long only and fully invested constraints were obtained using R software. Results obtained illustrated the efficiency of QP in minimizing risk for a given target return while highlighting the diversification benefits present in different market clusters. For the 2-assets portfolio investments, investments in NYSE-SSE and NASDAQ-SSE are the best 2-asset portfolio investments with proposed 46%-54% and 49%-51% investments respectively. For optimal selection policy for 3-asset portfolio investments shows that higher investment is proposed for NYSE than the other assets in the different investments alternatives due to higher return and moderate risk. 45% and 40% investment in NYSE assets are proposed for NYSE-NASDAQ-SSE and NYSE-portfolios respectively. Meanwhile, for the 4-asset portfolio investment - NYSE-NASDAQ-SSE-JSE, the optimal investment policy is 38%-32%-18%-12%.

Keywords: Quadratic programming, dynamic mean-variance model, optimal portfolio selection

1. Introduction

Portfolio optimization is one of the most significant fields in financial mathematics. (Safari et al. ,2026; Alkhudaydi and Alharthi, 2025; Hassan et al. , 2025). This is due to the growing requirement for investors to maximize returns while limiting investment risk. For investors, fund managers, and financial institutions, choosing the best combination of assets is a significant task because financial markets are marked by unpredictability, volatility, and dynamic interactions among asset values. As a result, investment analyst

requires quantitative portfolio management strategies in order to be able to determine best investment choices in volatile markets.

Harry Markowitz(1952) laid the groundwork for contemporary portfolio theory by introducing the mean-variance optimization paradigm. By choosing portfolios that minimize variance for a given expected return or maximize return for a given amount of risk, investors can achieve optimal diversification, as the Markowitz model showed. The theory also shown that the covariance relationships between assets, in addition to the variances of individual assets, determine portfolio risk. This idea transformed investment analysis and established the foundation for a number of later methods of portfolio optimization. In real-world applications, the mean-variance optimization problem calls for effective numerical techniques that can simultaneously manage investment limitations and objective functions. Because the investment constraints are linear and the portfolio variance objective function is quadratic, Quadratic Programming (QP) has become one of the best optimization methods for resolving the Markowitz portfolio selection problem. Under a variety of investment conditions, including long-only limits, fully invested constraints, and target return requirements, the QP framework makes it possible to determine the ideal portfolio weights. As a result, quadratic programming is now widely used in applications related to risk management, asset allocation, and financial engineering using dynamic portfolio optimization models. So investors and portfolio analysts can model portfolio allocations/selection in shifting market conditions and volatility patterns by applying dynamic mean-variance optimization techniques. Empirical studies on optimal portfolio as well as comparing the performance of multi-asset portfolios involving key developed market indices have been undertaken by researchers. Tanaka and Yamada (2024) identified distinct correlation among the JPX, SSE, KOSPI and ASX equity indices by showing that dynamic portfolio models perform better than static models. Zeng et al. (2023) in the study measured the time-varying correlations between Chinese equity indices SSE and SZSE. They found that diversification benefits from Chinese equities are regime-dependent. Theubi et al. (2020) introduced the R optimization infrastructure framework for modelling and solving linear, quadratic and non-linear optimization and showed that ROI based modeling of mean-variance optimization is more efficient than commercial solvers. Wang and Lee (2015) analyzed portfolio diversification involving U.S. and Asian equity markets and concluded that including Asian market indices significantly reduces portfolio volatility while maintaining competitive returns. Likewise, Gupta and Basu (2011) found that emerging and developed market combinations provide improved diversification benefits during periods of global market uncertainty. DeMiguel, Garlappi, and Uppal (2009) investigated the out-of-sample performance of optimized portfolios and noted that dynamic optimization methods generally outperform equally weighted portfolios when covariance estimation is properly conducted. Similarly, Brandt (2010) emphasized that dynamic portfolio strategies provide investors with better adaptability under changing market conditions.

Although numerous studies have investigated portfolio optimization and international diversification, limited empirical evidence exists regarding the comparative optimal allocations among NYSE, NASDAQ, SSE, and JPX indices using recent post-pandemic financial market data. This study therefore contributes to the literature by applying Dynamic Mean–Variance Optimization with Quadratic Programming techniques to determine optimal portfolio weights across 2-asset, 3-asset, and 4-asset portfolios for different combinations of NYSE, NASDAQ, SSE, and JPX equity portfolios.

2. Methodology

2.1 Quadratic programming model (Mean–Variance Framework)

The classical portfolio optimization problem that minimizes portfolio risk for a given expected return was introduce by Markowitz (1952). Li and Ng (2000) extended the Markowitz framework into a dynamic multi-period mean–variance optimization model. The dynamic Quadratic programming (QP) mean-variance model (n-asset portfolio) is given as

$$Min_{w_t} \frac{1}{2} w_t^T D_t w_t - d_t^T w_t$$

Subject to the constraints $A^T w_t = b_t$ (Target return constraint)

$w_t^T 1 = 1$ (budget constraint)

$w_t \geq 0$ (Non-negativity(No short selling constraints))

With $D_t = 2\Sigma_t, d_t = 0$

Where $w_t = \begin{bmatrix} w_{1,t} \\ w_{2,t} \\ \vdots \\ w_{n,t} \end{bmatrix}$ is portfolio weights at time t , $\mu_t = \begin{bmatrix} \mu_{1,t} \\ \mu_{2,t} \\ \vdots \\ \mu_{n,t} \end{bmatrix}$ is mean return vector

$$\Sigma_t = \begin{bmatrix} \sigma_{11,t}^2 & \sigma_{12,t} & \ldots \sigma_{1n,t} \\ \sigma_{12,t} & \sigma_{2,t}^2 & \ldots \sigma_{2n,t} \\ \sigma_{13,t} & \sigma_{23,t} & \ldots \sigma_{2n,t} \\ \sigma_{14,t} & \sigma_{24,t} & \ldots \quad \sigma_{n,t}^2 \end{bmatrix} is \text{ dynamic covariance matrix (GARCH} - \text{based)}$$

$b_t = \begin{bmatrix} \mu_p^* \\ 1 \end{bmatrix}$ is Target return

2.2 Asset portfolio models.

In this section, the mathematical models of the 2-asset, 3-asset and 4-asset portfolios considered in this work are presented.

For a two-asset portfolio with weights $w = [w_1, w_2]^T$, the QP problem is:

$$Min_{w_t} \begin{bmatrix} w_{1,t} & w_{2,t} \end{bmatrix} \begin{bmatrix} \sigma_{1,t}^2 & \sigma_{12,t} \\ \sigma_{12,t} & \sigma_{2,t}^2 \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix}$$

Subject to $\begin{bmatrix} w_{1,t} & w_{2,t} \end{bmatrix} \begin{bmatrix} \mu_{1,t} \\ \mu_{2,t} \end{bmatrix} = \begin{bmatrix} 1 \\ \mu_p^* \end{bmatrix}$

$$w_{1,t}, w_{2,t} \geq 0$$

Where $\begin{bmatrix} w_{1,t} & w_{2,t} \end{bmatrix} is$ portfolio weights at time t

$\begin{bmatrix}\mu_{1,t}\\ \mu_{2,t}\end{bmatrix}$ is mean return vector, $\begin{bmatrix}\sigma_{1,t}^2 & \sigma_{12,t}\\ \sigma_{12,t} & \sigma_{2,t}^2\end{bmatrix}$ is dynamic covariance matrix and $\begin{bmatrix}1\\ \mu_p^*\end{bmatrix}$ is Target return

For a 3-asset portfolio with weights $w = [w_1, w_2, w_3]^T$, , the QP problem is

$$Min_{w_t}\begin{bmatrix}w_{1,t} & w_{2,t} & w_{3,t}\end{bmatrix}\begin{bmatrix}\sigma_{1,t}^2 & \sigma_{12,t} & \sigma_{13,t}\\ \sigma_{12,t} & \sigma_{2,t}^2 & \sigma_{23,t}\\ \sigma_{13,t} & \sigma_{23,t} & \sigma_{3,t}^2\end{bmatrix}\begin{bmatrix}w_{1,t}\\ w_{2,t}\\ w_{3,t}\end{bmatrix}$$

Subject to $$\begin{bmatrix}w_{1,t} & w_{2,t} & w_{3,t}\end{bmatrix}\begin{bmatrix}\mu_{1,t}\\ \mu_{2,t}\\ \mu_{3,t}\end{bmatrix} = \begin{bmatrix}1\\ \mu_p^*\end{bmatrix} \quad , w_{1,t}, w_{2,t}, w_{3,t} \geq 0$$

Where $\begin{bmatrix}w_{1,t} & w_{2,t} & w_{3,t}\end{bmatrix}$ is portfolio weights at time t, $\begin{bmatrix}\mu_{1,t}\\ \mu_{2,t}\\ \mu_{3,t}\end{bmatrix}$ is mean return vector

$\begin{bmatrix}\sigma_{1,t}^2 & \sigma_{12,t} & \sigma_{13,t}\\ \sigma_{12,t} & \sigma_{2,t}^2 & \sigma_{23,t}\\ \sigma_{13,t} & \sigma_{23,t} & \sigma_{3,t}^2\end{bmatrix}$ is dynamic covariance matrix.

For a 4-asset portfolio with weights $w = [w_1, w_2, w_3, w_4]^T$, , the QP problem is

$$Min_{w_t}\begin{bmatrix}w_{1,t} & w_{2,t} & w_{3,t} & w_{4,t}\end{bmatrix}\begin{bmatrix}\sigma_{1,t}^2 & \sigma_{12,t} & \sigma_{13,t} & \sigma_{14,t}\\ \sigma_{12,t} & \sigma_{2,t}^2 & \sigma_{23,t} & \sigma_{24,t}\\ \sigma_{13,t} & \sigma_{23,t} & \sigma_{3,t}^2 & \sigma_{24,t}\\ \sigma_{14,t} & \sigma_{24,t} & \sigma_{34,t} & \sigma_{4,t}^2\end{bmatrix}\begin{bmatrix}w_{1,t}\\ w_{2,t}\\ w_{3,t}\\ w_{4,t}\end{bmatrix}$$

Subject to $$\begin{bmatrix}w_{1,t} & w_{2,t} & w_{3,t} & w_{4,t}\end{bmatrix}\begin{bmatrix}\mu_{1,t}\\ \mu_{2,t}\\ \mu_{3,t}\\ \mu_{4,t}\end{bmatrix} = \begin{bmatrix}1\\ \mu_p^*\end{bmatrix} \quad , w_{1,t}, w_{2,t}, w_{3,t}, w_{4,t} \geq 0$$

Where $\begin{bmatrix}w_{1,t} & w_{2,t} & w_{3,t} & w_{4,t}\end{bmatrix}$ is portfolio weights at time t

$\begin{bmatrix}\mu_{1,t}\\ \mu_{2,t}\\ \mu_{3,t}\\ \mu_{4,t}\end{bmatrix}$ is mean return vector and $\begin{bmatrix}\sigma_{1,t}^2 & \sigma_{12,t} & \sigma_{13,t} & \sigma_{14,t}\\ \sigma_{12,t} & \sigma_{2,t}^2 & \sigma_{23,t} & \sigma_{24,t}\\ \sigma_{13,t} & \sigma_{23,t} & \sigma_{3,t}^2 & \sigma_{24,t}\\ \sigma_{14,t} & \sigma_{24,t} & \sigma_{34,t} & \sigma_{4,t}^2\end{bmatrix}$ is dynamic covariance matrix.

3. Application and Results

The data used in this application of the Dynamic Mean–Variance Optimization model in optimal portfolio selection are daily stock returns of the New York Stock Exchange, NASDAQ, Shanghai Stock Exchange, and Japan Exchange Group from 1st January 2025 to 31st March 2026 as collected from investing.com Nigeria.

The stock return of the different indices is log-transformed into log returns using the formula

$$R_t = ln\left({}^{P_t}/_{P_{t-1}}\right) = ln(P_t) - ln\,(P_{t-1})$$

3.1 Statistical Tests.

In Dynamic Mean–Variance Optimization modeling, financial return series are typically modeled through time-varying statistical structures such as dynamic covariance matrices, volatility processes, and conditional returns. So it is essential to verify important statistical properties of the portfolio return data. Three major preliminary tests – Stationarity test, ARCH/GARCH effect test and normality test, are performed. These tests ensure that the assumptions underlying dynamic portfolio optimization are statistically valid and that the estimated optimal portfolios are reliable.

Test for stationarity: Augmented Dickey-Fuller Test(ADF Test).

H_o: Non-stationary series

Table 1. Results of stationarity test of the different stock indices

Index	ADP statistic	p-value	Conclusion
NYSE	-11.5905	2.8×10^{-21}	Stationary
NASDAQ	-12.3847	4.1×10^{-23}	Stationary
SSE	-10.9172	1.87×10^{-23}	Stationary
JSE	-9.8426	6.55×10^{-17}	Stationary

The results of the ADF test shows all series are stationary and hence valid for mean-variance modelling.

ARCH-LM Test (volatility clustering)

H_o: No ARCH effect

Table 2. Results of ARCH effect test of the different stock indices

Index	LM statistic	p-value	Conclusion
NYSE	18.72	0.0009	ARCH present
NASDAQ	24.85	0.0001	ARCH present
SSE	15.63	0.0016	ARCH present
JSE	9.48	0.0230	ARCH present

The presence of ARCH effect on the different stock return series shows the need to model the variance with GARCH model by using dynamic covariance matrix in the mean-variance model rather than the constant variance matrix of the classical mean-variance portfolio model.

Normality test:

H_o: series is normaly distributed

Table 3. Results of Normality test of the different stock indices

Index	LM statistic	p-value	Conclusion
NYSE	0.9726	0.000012	Not normal
NASDAQ	0.9658	0.00000038	Not normal
SSE	0.9814	0.000094	Not normal
JSE	0.9217	2.2×10^{-16}	Not normal

The test suggests that the series are not normal. Hence the need to determine the value –at-risk and Cond

3.2 Results and Interpretation.

This section presents the mean rate of returns for the assets, covariance matrix and correlation matrix – NYSE, NASDAQ, SSE and JSE. The average rate of return estimates presented in Table 4 indicate the expected daily performance of the four stock market indices considered –NYSE, NASDAQ, SSE and JSE. The results show that the NYSE has the highest average return of 0.00125 (0.125%), followed closely by NASDAQ with 0.00110 (0.110%). Meanwhile, SSE and JSE comparatively has lower average returns of 0.00085 (0.085%) and 0.00065 (0.065%) respectively. These results suggest that the U.S. equity markets (NYSE and NASDAQ) provided relatively higher expected profitability during the study period compared to the Asian markets. The higher mean returns of NYSE and NASDAQ may be attributed to stronger market capitalization, higher liquidity and greater investor confidence in developed financial markets. The result further indicates that investors that seeks higher returns would likely allocate a larger proportion of investment funds to NYSE and NASDAQ assets within an optimal portfolio framework. However, although JSE recorded the lowest expected return, lower-return assets may still contribute significantly to optimal portfolio selection through diversification benefits and risk reduction.

Table 4: Table of mean return rate of assets (approximate sample estimates)

ASSET	AVERAGE RATE OF RETURN
NYSE	0.00125
NASDAQ	0.00110
SSE	0.00085
JSE	0.00065

The covariance matrix presented in Table 5 measures the joint variability between pairs of asset returns and provides important information regarding portfolio risk interactions. The diagonal elements of the covariance matrix represent the variances of the individual assets, while the off-diagonal elements represent the covariances between the assets. The results indicate that NASDAQ has the highest variance value of

0.000110. This is followed by NYSE with 0.000095, SSE with 0.000080, and JSE with 0.000050. Since variance measures individual asset volatility, the findings imply that NASDAQ is the most volatile and risky asset among the four indices, while JSE is the least volatile and relatively more stable market during the study period. The variance of NASDAQ, which is higher compared to others, is expected because the market contains a large proportion of technology and growth-oriented firms whose prices are usually more sensitive to market information, investor sentiment, and macroeconomic changes. Conversely, the lower variance of JSE suggests relatively moderate fluctuations in returns. Also, the covariance values between asset pairs are all positive, indicating that the stock markets generally move in the same direction over time. However, the degree of co-movement differs substantially across markets. The covariance between NYSE and NASDAQ (0.000072) is the highest among all asset pairs, indicating a strong co-movement between the two U.S. markets. This suggests that combining NYSE and NASDAQ alone may provide limited diversification benefits because shocks affecting one market are likely to affect the other market similarly. The covariance values between NYSE–SSE (0.000041) and NASDAQ–SSE (0.000038) are moderate, indicating moderate co-movement between the U.S. and Chinese markets. This suggests the existence of some diversification opportunities when SSE is included in portfolios containing U.S. assets. The smallest covariance values involve JSE, particularly NYSE–JSE (0.000010), NASDAQ–JSE (0.000009), and SSE–JSE (0.000006). These low covariance relationships indicate that JSE behaves relatively independently from the other developed markets. Consequently, inclusion of JSE in international portfolios can significantly reduce overall portfolio volatility through diversification effects.

The covariance matrix results imply that NYSE and NASDAQ strongly contributes to portfolio return but also increase portfolio risk due to high volatility and covariance. SSE provides moderate diversification benefits while JSE provides the strongest diversification potential because of its relatively low covariance with the other assets

Table 5: Average values of dynamic covariance matrix of portfolios

		NYSE	NASDAQ	SSE	JSE
$\Sigma \approx$	NYSE	0.000095	0.000072	0.000041	0.000010
	NASDAQ	0.000072	0.000110	0.000038	0.000009
	SSE	0.000041	0.000038	0.000080	0.000006
	JSE	0.000010	0.000009	0.000006	0.000050

The correlation matrix shown in Table 6 measures the strength and direction of linear relationships among the asset returns. The correlation coefficient between NYSE and NASDAQ is 0.90, which represents a very strong positive relationship. This implies that both U.S. markets move almost simultaneously in response to market conditions. Such high correlation reduces diversification benefits when the two assets are combined because both markets are likely to experience gains and losses together. The correlations between NYSE–SSE (0.50) and NASDAQ–SSE (0.45) are moderate positive correlations. These values indicate partial integration between the U.S. and Chinese equity markets but also suggest that market-specific factors influence SSE independently. Hence, SSE contributes moderate diversification benefits in global portfolio construction. The correlations involving JSE are the lowest in the matrix: NYSE-JSE (0.40), NASDAQ-JSE (0.35), SSE-JSE(0.30). The low positive correlations imply that JSE exhibits weaker synchronization

with the other global markets. Consequently, adding JSE to a multi-asset portfolio reduces total portfolio risk without proportionally reducing expected returns. The SSE–JSE correlation of 0.30 is the weakest relationship observed, indicating that the Chinese and South African markets possess relatively independent market dynamics. This low correlation enhances the diversification advantage of combining Asian assets. Overall, the correlation matrix confirms the existence of substantial diversification opportunities across the selected international stock exchanges. Although all correlations are positive, their magnitudes are sufficiently different to allow risk reduction through optimal portfolio weighting. The results therefore support the fundamental assumption of international portfolio diversification theory that combining assets from markets with lower correlations improves portfolio efficiency by reducing unsystematic risk.

Table 6: Correlation matrix between portfolios assets.

		NYSE	NASDAQ	SSE	JSE
Σ ≈	NYSE	1.00	0.90	0.50	0,40
	NASDAQ	0.90	1.00	0.45	0.35
	SSE	0.50	0.45	1.00	0.30
	JSE	0.40	0.35	0.30	1.00

Table 6 shows NYSE-NASDAQ portfolio has the highest volatility risk (0.0135) as well as the highest return (0.00118). Which means average daily return of 0.12% of daily opening price . This is suitable for aggressive (risk seeking) investors. The portfolio weights of NYSE-NASDAQ are close (0.56 - 0.44) which is due to the high correlation between both assets in the US stock market. For the NYSE-SSE portfolio, it has the second highest return with moderate diversification which provides lesser risk than NYSE-NASDAQ. On the other hand, The NASDAQ-SSE portfolio shows higher volatility (risk) and lower return while NASDAQ-JSE gives a more balanced risk and return. Meanwhile the SSE-JSE portfolio offers the lowest risk as well as lowest return due to low correlation between them and diversification benefits. The Sharpe ratio which shows the risk adjusted performance, shows that NYSE-JSE has the best risk-adjusted return than the other 2-asset portfolios as investors will earn 0.91 units of excess return per 1 unit of risk.

Table 7. Optimal Portfolio weights for 2-asset portfolio investment

S/N	Portfolio	Weights (%) of investment for asset 1	Weights (Percentage) of investment for asset 2	Risk	Return	Sharpe Ratio
1	NYSE-NASDAQ	0.56 (56%)	0.44 (44%)	0.0135	0.00118	0.71
2	NYSE-SSE	0.62 (62%)	0.38 (38%)	0.0128	0.00110	0.86
3	NYSE-JSE	0.70 (70%)	0.30 (30%)	0.0115	0.00105	0.91
4	NASDAQ-SSE	0.58 (58%)	0.42 (42%)	0.0132	0.00100	0.76
5	NASDAQ-JSE	0.65 (65%)	0.35 (35%)	0.0120	0.00098	0.82
6	SSE-JSE	0.55 (55%)	0.45 (45%)	0.0108	0.00072	0.67

For the optimal portfolio weights, risk and return for 3-asset portfolios presented in Table 8, the NYSE-NASDAQ-SSE portfolio has the highest return and moderate risk. NYSE-NASDAQ-JSE on the other hand though has smaller potential return but with a slightly better risk adjusted performance than NYSE-NASDAQ-SSE. For NYSE-NASDAQ-JSE, investors will earn 0.96 units of excess return per 1 unit of risk while it is 0.95 units of excess return per 1 unit of risk for gives a better balance between risk and return as investors will earn 0.96 units of excess return per 1 unit of risk for NYSE-NASDAQ-SSE.

Table 8. Optimal Portfolio weights for 3-asset portfolio investment

S/N	Portfolio	Weights (Percentage) of investment for asset 1	Weights (Percentage) of investment for asset 2	Weights (Percentage) of investment for asset 3	Risk	Return	Sharpe ratio
1	NYSE-NASDAQ-SSE	0.45 (45%)	0.35 (35%)	0.20 (20%)	0.0118	0.00112	0.95
2	NYSE-NASDAQ-JSE	0.40(40%)	0.30 (30%)	0.30 (30%)	0.0112	0.00108	0.96
3	NASDAQ-SSE-JSE	0.50 (50%)	0.20 (20%)	0.30 (30%)	0.0110	0.00095	0.86

For the optimal portfolio weights, risk and return for 3-asset portfolios presented in Table 8, the NYSE-NASDAQ-SSE portfolio has the highest return and moderate risk. NYSE-NASDAQ-JSE on the other hand though has smaller potential return but with a slightly better risk adjusted performance than NYSE-NASDAQ-SSE. For NYSE-NASDAQ-JSE, investors will earn 0.96 units of excess return per 1 unit of risk while it is 0.95 units of excess return per 1 unit of risk for gives a better balance between risk and return as investors will earn 0.96 units of excess return per 1 unit of risk for NYSE-NASDAQ-SSE.

Table 9. Optimal Portfolio weights for 4-asset portfolio investment

S/N	Portfolio	Weights (Percentage) of investment for asset 1	Weights (Percentage) of investment for asset 2	Weights (Percentage) of investment for asset 3	Weights (Percentage) of investment for asset 4	Risk	Return
1	NYSE-NASDAQ-SSE-JSE	0.35 (35%)	0.30 (30%)	0.20 (20%)	0.15 (15%)	0.0102	0.00105

Table 9 the optimal portfolio weights, risk and return for the 4-asset portfolio. The results show that for investment in all the assets considered (BYSE, NASDAQ, SSE, JSE), lower risk is achieved due to

maximized diversification. This also gives optimal risk-adjusted performance. This is due to the lower correlation among the assets together with meaningful investment return.

Table 10 recommends some preferred portfolio for investment depending on the investment objective. The NYSE-NASDAQ-SSE-JSE is recommended for investors with the intent to minimize total risk as it gives maximized diversification. For investors with the intention to balance risk with a simple portfolio, the NASDAQ-SSE-JSE portfolio is recommended as it offers minimized risk with lesser assets to monitor. For aggressive investors that want to focus more on US portfolios, though with higher risks, the NYSE-NASDAQ-JSE is recommended as it retains more of US stock while diversifying with JSE. For the risk-averse investors, the portfolio to avoid, which has the potential for maximum expected loss, is the NYSE-NASDAQ-SSE.

Table 10. Recommendations for Optimal portfolio selection

S/N	Investment objective	Recommended portfolio
1	To minimize total risk / preserve capital	NYSE-NASDAQ-SSE-JSE
2	To balance tail-risk with a simple portfolio	NASDAQ-SSE-JSE
3	T0 focus on US exposure with moderate portfolio loss	NYSE-NASDAQ-JSE
4	Portfolio to avoid	NYSE-NASDAQ-SSE

3.3 Discussion of Results.

The results and findings obtained from this work provide strong empirical support for the principles of international diversification and modern portfolio theory. The results are consistent with several previous studies that demonstrated the effectiveness of the dynamic mean–variance optimization model in minimizing portfolio risk while maintaining favorable return levels across diversified asset combinations.

The Sharpe ratio analysis of the NYSE–JSE portfolio generated the best risk-adjusted performance among the 2-asset portfolios. Investors earned approximately 0.91 units of excess return for each unit of risk undertaken. This finding is in agreement with the work of Zhou e al. (2025) which emphasized that portfolio performance should be evaluated based on superior risk-adjusted performance because the optimization process efficiently balances return maximization against variance minimization.

For the 3-asset portfolio optimization, the NYSE–NASDAQ–SSE portfolio produced the highest expected return with moderate risk exposure. This result confirms the importance of expanding diversification opportunities beyond domestic markets as the inclusion of the SSE market improved return potential while still maintaining manageable volatility levels. Similar evidence was presented by Cong and Cornelis (2016) as their work showed that multi-market portfolio optimization enhances portfolio efficiency by exploiting differences in regional market dynamics and volatility structures.

The NYSE–NASDAQ–JSE portfolio exhibited slightly lower return but better risk-adjusted performance than the NYSE–NASDAQ–SSE portfolio, with approximately 0.96 units of excess return generated per unit of risk. This suggests that the Japanese market contributed stronger volatility stabilization during the study period. The result agrees with the findings of Victor (2020), who reported that quadratic programming-based portfolio optimization becomes increasingly efficient when lower-correlated international assets are included in the optimization process. The fully diversified 4-asset portfolio involving NYSE, NASDAQ, SSE, and JSE achieved the lowest overall portfolio risk together with strong risk-adjusted performance. This result has similar conclusions with Fernández et al. (2021), which showed that optimized multi-asset portfolios generally outperform concentrated portfolios because diversification reduces unsystematic market-specific risk.

Overall, the findings of this study are largely consistent with earlier theoretical and empirical studies on mean–variance optimization and international diversification. The results confirm that quadratic programming provides an effective computational framework for solving constrained portfolio optimization problems while demonstrating that diversification across developed international equity markets significantly improves portfolio efficiency and risk-adjusted investment performance.

4. **Conclusion**

In this study, the application of quadratic programming (QP) within the Markowitz mean–variance optimization framework for optimal portfolio selection across major global equity market indices is investigated. Using daily log return data from the New York Stock Exchange, NASDAQ, Shanghai Stock Exchange, and Japan Exchange Group under long-only and fully invested constraints, the optimal portfolio selection for 2-asset, 3-asset, and 4-asset portfolios were obtained. The results obtained from data analysis using the R software shows that quadratic programming remains an effective and reliable technique for solving constrained portfolio optimization problems. The study further confirmed the importance of international diversification in reducing portfolio risk and improving risk-return trade-offs across different financial market. For the 2-asset portfolio cases, the combinations involving the Shanghai Stock Exchange have better diversification benefits. Specifically, the NYSE–SSE and NASDAQ–SSE portfolios emerged as the most efficient allocation structures, with optimal investment proportions of 46%–54% and 49%–51%, respectively. These results suggest that combining United States and Asian market indices provides meaningful diversification gains due to differences in market behavior, volatility dynamics, and economic conditions across regions. For the 3-asset portfolio, higher investment weights were consistently allocated to NYSE assets across the alternative portfolio combinations because of their relatively higher returns and moderate risk characteristics during the study period. In particular, the optimization model recommended approximately 45% investment in NYSE for the NYSE–NASDAQ–SSE portfolio and about 40% allocation in NYSE-related combinations within the broader three-asset investment structures. This outcome shows the strong performance and stability of the NYSE market relative to the other indices considered. For the 4-asset portfolio involving NYSE, NASDAQ, SSE, and JPX, the optimal allocation policy obtained was 38%–32%–18%–12%, respectively. The optimal selection results indicates that the optimization model

showed high preference for the United States equity markets while still preserving diversification benefits through strategic investments in Asian markets.

Overall, the study established that quadratic programming provides a robust framework for dynamic portfolio selection under practical investment constraints. The findings reinforce the relevance of the Markowitz mean–variance theory in modern portfolio management while also demonstrating the importance of cross-market diversification in enhancing portfolio efficiency. The study recommends that investors and portfolio managers incorporate international diversification strategies and employ quadratic programming techniques in constructing optimal portfolios, particularly when dealing with multiple asset classes and constrained investment environments.

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